1. Consider the following differentials:

$$\begin{aligned} đu &= -\frac{y}{x^2+y^2}dx + \frac{x}{x^2+y^2}dy \\ đu &= \frac{y}{z}dx + \frac{x}{z}dy - \frac{xy}{z^2}dz \\ đu &= (2y^2 - 3x)dx - 4xy\,dy \\ đu &= (x + y/x)dx + dy \end{aligned}$$

   Which of these are exact, and in case of inexact differentials, find the integrating factor.

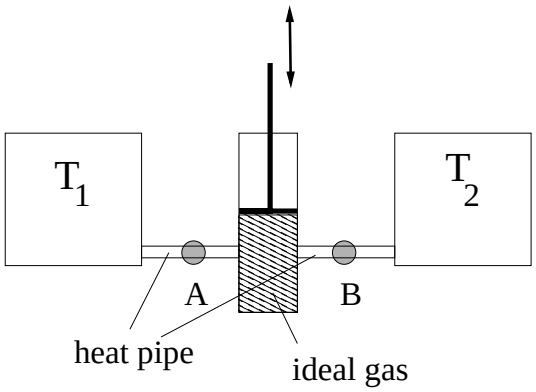
2. Calculate the Legendre transformation from variable x to y; $f(x) \to g(y) = f(x(y)) - xy$, $y = \partial f/\partial x$, when

   a) $f(x) = e^x$

   b) $f(x) = x^n$ for integer n

3. Calculate the virial expansion coefficients for the Van der Waals equation of state. Show that the second coefficient is $B_2(T) = b - \frac{a}{k_B T}$.

4. A realization of the Carnot engine: consider a piston filled with ideal gas connected to 2 heat reservoirs at temperatures T_1 and T_2, with $T_2 > T_1$. The connection is via "heat pipes", which are equipped with switches A and B. When both switches are disconnected the piston is thermally isolated.



   What are the 4 stages of the Carnot cycle in this setup? Let us assume that during the isothermal stage at $T = T_2$ (stage a in the lecture notes) the volume of the system expands from $V_1 \to V_2$. Calculate ΔW, ΔQ and ΔV during the 4 stages of the cycle.

   You will need the ideal gas equation of state, $pV = Nk_BT$, and the expression for the internal energy $U = \frac{3}{2}Nk_BT$ (for monatomic ideal gas).