Statistical physics

1. Calculate the equation of state, entropy and heat capacity for the dilute $(n = N/V \rightarrow 0)$ Bose gas taking into account only leading deviations from the classical ideal gas. Note that in this limit $\mu \rightarrow -\infty$ and fugasity $z = e^{\beta\mu}$ is small and proportional to n. The results for 2nd virial coefficient, entropy and C_V are

$$B_2(T) = \frac{\hbar^3}{2g} \left(\frac{\pi}{mk_BT}\right)^{3/2}$$

$$S = S^{\text{class.}} - Nk_B \frac{n\hbar^3}{4g} \left(\frac{\pi}{mk_BT}\right)^{3/2}$$

$$C_V = C_V^{\text{class.}} + Nk_B \frac{3n\hbar^3}{8g} \left(\frac{\pi}{mk_BT}\right)^{3/2}$$

At which temperature does n deviate 10% from the classical value at atmospheric pressure for bosons with a mass of ${}^{4}He$ -atom?

2. Show the following results for the ideal Bose gas below the condensation temperature:

$$E = c_1 m^{3/2} T^{5/2} V$$

$$p = c_2 m^{3/2} T^{5/2}$$

$$C_V = \frac{5}{2} \frac{E}{T}$$

$$S = \frac{5}{3} \frac{E}{T}$$

What are constants c_1, c_2 ?

3. Determine the so-called solar constant, which is the solar radiation power measured from a surface element perpendicular to the direction to the Sun (W/m²). The surface temperature of the Sun is $T_S \approx 5800 \,\mathrm{K}$, diameter $1.392 \times 10^9 \,\mathrm{m}$ and mean distance from Earth $1.496 \times 10^8 \,\mathrm{km}$. Compare to the experimental value $1.39 \,\mathrm{kW/m^2}$