

1. Calculate the surface temperature of the Earth by assuming that the Earth is a black body with a constant surface temperature in radiation balance with the Sun. The diameter of the Sun is 1.4×10^6 km, distance from Earth 1.5×10^8 km and surface temperature 5800 K.
2. The energy and entropy for a non-rotating black hole of mass M can be defined as

$$E = Mc^2 \quad S = \frac{k_B A}{8\pi\ell_P^2}$$

where $A = 4\pi R^2$ is the surface area of the black hole, with $R = 2GM/c^2$ the Schwarzschild radius, and $\ell_P = \sqrt{\hbar G/c^3}$ is the Planck length, and the Newton constant $G = 6.673 \times 10^{-11} \text{ Nm}^2/(\text{kg})^2$.

- Calculate the temperature of the black hole as a function of mass using thermodynamical identities.
 - Black hole radiates like an (almost) perfect black body (why?). Calculate the radiation power P and the timescale associated with the loss of mass, $\tau = E/P$.
 - Calculate the temperature T and lifetime τ in years for a black hole of mass 10^{12} kg.
 - Currently the Universe is filled with radiation of $T \approx 2.7$ K, as a remnant of the big bang. How does this affect the development of the above black hole (qualitatively)?
3. Does Bose condensation occur for 2-dimensional ideal BE gas?
 4. Show that the ideal (non-interacting) gas undergoing adiabatic process obeys

$$VT^\alpha = \text{constant}, \quad pV^\gamma = \text{constant}, \quad \frac{T^{\alpha\gamma}}{p} = \text{constant}.$$

Calculate α and γ for non-relativistic MB, BE and FD gas.