1. The thermal expansion coefficient and compressibility are defined as

$$\alpha_p = \frac{1}{V}\left(\frac{\partial V}{\partial T}\right)_p ; \kappa_T = -\frac{1}{V}\left(\frac{\partial V}{\partial p}\right)_T .$$

   a) Show that

$$\left(\frac{\partial \alpha}{\partial p}\right)_T = -\left(\frac{\partial \kappa}{\partial T}\right)_p \quad \text{and} \quad \frac{\alpha}{\kappa} = \left(\frac{\partial p}{\partial T}\right)_V .$$

   b)Calculate α, κ and $(\partial p/\partial T)_V$ for the ideal gas.

2. Show that

$$\kappa_T = \kappa_S + VT\frac{\alpha_p^2}{C_p}.$$

3. Let us consider homogeneous magnetic system, with internal energy $U = U(S, M)$ and $dU = TdS + \mu_0 HdM$, where H is the magnetic field strength and M the total magnetization. Calculate the expressions for the thermodynamic potentials U, F, G, E (here $E =$ enthalpy), and for the differentials thereof.

   Express the heat capacities

$$C_M = T\left(\frac{\partial S}{\partial T}\right)_M \qquad C_H = T\left(\frac{\partial S}{\partial T}\right)_H$$

   and susceptibilities

$$\chi_T = \left(\frac{\partial M}{\partial H}\right)_T \qquad \chi_S = \left(\frac{\partial M}{\partial H}\right)_S$$

   in terms of the thermodynamic potentials.

4. Show that

$$\left(\frac{\partial U}{\partial Y}\right)_T = T\left(\frac{\partial X}{\partial T}\right)_Y + Y\left(\frac{\partial X}{\partial Y}\right)_T ,$$

   where Y is a generalized force and X displacement.