1. Write the van der Waals equation of state in terms of molecular volume $v = V/N$ (inverse of density). Sketch the behaviour of isotherms $p(v, T = \text{const.})$ at large and small T. What is the critical point (T_c, p_c, v_c)?

2. a) Calculate the phase diagram of van der Waals gas on (T, p)-plane in close proximity of the critical point (i.e. deviations from the critical quantitites are small).

   b) Find the discontinuity of the density $n = N/V$ near the critical point, i.e. find $\Delta n = n_+(p, T) - n_-(p, T)$, where $n_\pm$ are the densities of liquid and gas along the coexistence line. If we write $\Delta n \propto (T_c - T)^\beta$, find the value of the exponent β.

   c) Find the latent heat ΔH near the critical point.

3. Show that the ensemble probability density function ρ obeys the Liouville equation

$$i\frac{\partial \rho}{\partial t} = L\rho\,,$$

where L is the Liouville operator

$$L = i\{H, \quad\} = i\sum_j \left(\frac{\partial H}{\partial q_j}\frac{\partial}{\partial p_j} - \frac{\partial H}{\partial p_j}\frac{\partial}{\partial q_j}\right).$$

If H is independent of t, show that the formal solution of the equation is

$$\rho(t) = e^{-iLt}\rho(0)\,.$$

4. Let us assume that the probability distribution depends on P only through Hamilton function H,

$$\rho(P) = \rho(H[P])\,.$$

Show that the distribution is stationary, $\partial\rho/\partial t = 0$, and that the expectation value of an observable f with no explicit time dependence is also stationary:

$$\frac{d\langle f\rangle}{dt} = 0.$$