Statistical physics Problem set 9 9.11.2006 14.15

1. Show that $pV = \frac{2}{3}E$ for ideal Fermi gas.

2. a) Show the following relations for ideal Bose-Einstein gas:

$$\begin{aligned}(\Delta n_\ell)^2 &\equiv \langle (n_\ell - \bar{n}_\ell)^2 \rangle = \bar{n}_\ell(1+\bar{n}_\ell) \\ (\Delta N)^2 &\equiv \langle (N - \bar{N})^2 \rangle = \bar{N} + \sum_\ell \bar{n}_\ell^2\end{aligned}$$

where $\bar{N} = \sum_\ell \bar{n}_\ell$ is the total number of particles.

b) Show for ideal Fermi-Dirac gas

$$\begin{aligned}(\Delta n_\ell)^2 &= \bar{n}_\ell(1-\bar{n}_\ell) \\ (\Delta N)^2 &= \sum_\ell \bar{n}_\ell(1-\bar{n}_\ell)\end{aligned}$$

3. Defining the function

$$g_x(z) = \sum_{n=1}^{\infty} \frac{z^n}{n^x}$$

(this gives the Riemann zeta-function as $g_x(1) = \zeta(x)$), show that the integral representation of g is

$$g_x(z) = \frac{1}{\Gamma(x)} \int_0^\infty dy \frac{y^{x-1}}{z^{-1}e^y - 1}.$$

Show that for ideal Bose-Einstein gas

$$\Omega(T, V, \mu) = k_B T \ln(1-z) - \frac{V k_B T}{\lambda_T^3} g_{5/2}(z)$$

and

$$N(T, V, \mu) = \frac{z}{1-z} + \frac{V}{\lambda_T^3} g_{3/2}(z),$$

where $z = e^{\beta\mu}$ and λ_T is the thermal deBroglie wavelength.